Continuous Improvement Toolkit

Capability Indices
- Capability Indices

- A statistical tool that compares the actual process performance to the performance standards or design specifications.
- A measure of how well the process output (VOP) meets the customer requirements (VOC).
- Design specifications often are expressed as:
  - A target or a nominal value.
  - A tolerance or an allowance above or below the nominal value.
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Why Process Capability?

- Provide a baseline measure of process performance.
- Monitors progress toward target.
- Gauges effectiveness of improvements.
- It is a key performance indicator (KPI) for Six Sigma projects.
Voice of the Customer:

I want 100% of products within these Specs.

Otherwise it will be defective product...

...I will raise a Complaint!

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- Capability Indices

Consequences of Defects:

- Scrap (Spoilage) is created.
- Rework is also created to correct the defect.
- Work that is required to adjust, correct, or modify the process.
- The customer wouldn’t be happy when he received the product (or service).
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- Question: What causes the variation?
- Answer:
  - Poor understanding.
  - Poor training.
  - Poor monitoring.
  - Poor procedures.
  - Poor decision making.

The less variability, the less frequently bad output is produced.
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How Do We Determine if the Process is Meeting Specifications?

- **Graphical:**
  - If the process spread is smaller than or within the specification spread, the process is able to meet the specification.

- **Statistical:**
  - We use **Capability Indices** which incorporate the process spread and the specification into a single number.

Specification spread is sometimes referred as **Tolerance**
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The Graphical Approach – We Use Histograms To:

- Compare process output against specification limits.
- Predict the percentage of “Out-of-Specification” production.
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- The specification is the criteria used to decide if variability is acceptable.

- **Specification limits** are the minimum and maximum values that are acceptable.

- If the process is **stable**, this does not mean that it's meeting the specifications.

- A process is **capable** if it has a distribution whose extreme values fall within the specifications limit.
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Measure of Variability:

- Where the output data shows a normal distribution, the process is described by:
  - The mean (x).
  - The standard deviation (s).
- A control chart analysis is used to determine whether the process is in statistical control.
- If the process is not in statistical control then capability has no meaning.
- The more data included the more precise is the result.
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Approach:
- Ensure that the process is in control (stable).
- Measure the variability of the process.
- Compare **graphically** that variability with a proposed specification (or product tolerance).
- Measure process capability using descriptive indices.
- If results are acceptable, monitor the output using the control charts, and document when necessary.
- If results are unacceptable, further explore the assignable causes to reduce the variation or centering the process distribution on the nominal value.
Assumptions:
- The process is stable over time.
- The data is normally distributed.

If the data is non-normal:
- Transform the data and use normal capability tools.
- Use a different distribution that models the data.
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- Can we park the vehicles with no problems?
- Capability Indices

**Capability Indices:**

- Describe the overall effectiveness of a process in meeting specific criteria in both the short and long term.

**Capability Indices includes:**

- Percentage out of specification.
- Part per million out of specification (PPM).
- Potential capability (Cp and Cpk).
- Actual capability (Pp and PpK).
- Sigma value (Sigma level / Z bench).
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Part per Million:

- **Example:** What is the percentage out of specification in terms of part per million assuming that n = 100?

- **Answer:** 60,000 part per million are out of specification.

But what if the process looks like this?
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Potential Capability (Cp & Cpk):

- Represent what the process would be capable of if it did not have shifts and drifts.
- Also known as “within” or “short-term” capability.
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Cp:

- An index used to assess the width of the process spread in comparison to the width of the specification.
- The Cp states how many times the process can fit inside the specification.
- A Cp of 1 indicates that the width of the process and the width of the specification are the same.

\[ Cp = \frac{\text{Allowed variation (spec.)}}{\text{Normal variation of the process}} \]

\[ Cp = \frac{|USL - LSL|}{6\sigma} \]
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- A Cp of 1.3 means the process can fit inside the specification 1.3 times.
- Sometimes a Cp can be greater than one and yet still has data outside the specification.
- Cp takes no account of process settings.
- Use Cpk to overcome this problem.
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- **Parking A**
  - \( Cp = 2 \)
  - **Very good**
  - Parking space double than vehicle

- **Parking B**
  - \( Cp = 1.33 \)
  - **Good**
  - Parking space bigger than vehicle

- **Parking C**
  - \( Cp = 1 \)
  - **Regular**
  - Parking space same size of vehicle

- **Parking D**
  - \( Cp = 0.67 \)
  - **Bad**
  - Parking space smaller than vehicle
- Capability Indices

What can we do to solve the problem?

Cp = 0.67
Bad
Parking space smaller than vehicle
- Capability Indices

Reduce the variability of the process  Or  Change the specifications
- Capability Indices

Cpk:

- Takes into account the center of the data relative to the specifications (as well as the process variation).
- A Cpk of less than one means that some of the data is beyond the specification limit.
- The larger the Cpk, the more central and within specification the data.
- Cpk is always smaller or equal to Cp.

\[
Cpk = \text{Min} \left[ \frac{USL - Xbar}{3\sigma} \right] \text{ OR } \left[ \frac{Xbar - LSL}{3\sigma} \right]
\]
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\[
C_{pk} = \text{Negative number}
\]

\[
C_{pk} = \text{Zero}
\]

\[
C_{pk} = \text{Between 0 and 1}
\]

\[
C_{pk} = 1
\]

\[
C_{pk} > 1
\]
- Capability Indices

### Parking A
- **Cp = 2**
- **Cpk = 1**
- **Regular**
- Space enough but vehicle not centered

### Parking B
- **Cp = 1.33**
- **Cpk = 0.67**
- **Bad**
- Parking space bigger than vehicle, but vehicle not centered

### Parking C
- **Cp = 1**
- **Cpk = 1**
- **Regular**
- Parking space same size than vehicle. Vehicle centered

### Parking D
- **Cp = 0.67**
- **Cpk = 0.45**
- **Very Bad**
- Parking space smaller than vehicle and vehicle not centered
- Capability Indices

- When Cp and Cpk are over 1.0, the process is capable.
- The goal is to reduce variation so that all of the points fit within the specification limits.

<table>
<thead>
<tr>
<th>Cp / Cpk</th>
<th>Sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>3</td>
</tr>
<tr>
<td>1.33</td>
<td>4</td>
</tr>
<tr>
<td>1.67</td>
<td>5</td>
</tr>
<tr>
<td>2.0</td>
<td>6</td>
</tr>
</tbody>
</table>

A company targeting five-sigma level will aim for Cpk = 1.67
### Capability Indices

**Example – Benchmarked Capability Indices of a Company:**

<table>
<thead>
<tr>
<th></th>
<th>Cp</th>
<th>Cpk</th>
<th>Pp</th>
<th>Ppk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unacceptable</td>
<td>&lt; 1.6</td>
<td>&lt; 1.3</td>
<td>&lt; 1.3</td>
<td>&lt; 1.0</td>
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<tr>
<td>Borderline</td>
<td>1.6 – 1.8</td>
<td>1.3 – 1.6</td>
<td>1.3 – 1.6</td>
<td>1.0 – 1.3</td>
</tr>
<tr>
<td>Acceptable</td>
<td>1.8 – 2.0</td>
<td>1.6 – 1.8</td>
<td>1.6 – 1.8</td>
<td>1.3 – 1.6</td>
</tr>
<tr>
<td>World Class</td>
<td>&gt; 2.0</td>
<td>&gt; 1.8</td>
<td>&gt; 1.8</td>
<td>&gt; 1.6</td>
</tr>
</tbody>
</table>
- Capability Indices

Actual Capability (Pp & Ppk):
- Represent the actual performance of the process incorporating all observed variation.
- They estimate total variability from all sources.
- Also known as “overall” or “long-term” capability.
- Reflects more truthfully the current performance of the process.

\[ Pp = \frac{|USL - LSL|}{6\sigma} \]

\[ Ppk = \text{Min} \left[ \frac{USL - \bar{X}}{3\sigma} \right] \text{ OR } \left[ \frac{\bar{X} - LSL}{3\sigma} \right] \]
- Capability Indices

**Actual capability (Pp & Ppk):**

- Based on total process variation, including:
  - The effects of sampling variation.
  - The variation due to special causes and common causes.
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**Sigma Level:**
- A metric that measures the level of performance of a process based on the number of Defects per Million Opportunities (DPMO).
- Helps to determine how close (or far) the process is from Six Sigma.
- Calculated using Z Value (Z Score) from the Z Table.
- A high Sigma Level indicates a high level of customer satisfaction.
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- Z Value is a measurement of a data sample's distance from the population mean, calculated in standard deviations.

```
<table>
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<tr>
<th>Z</th>
<th>0.0214</th>
<th>0.1359</th>
<th>0.3413</th>
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<th>0.1359</th>
<th>0.0214</th>
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<tbody>
<tr>
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<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>0.6827</td>
<td>0.9545</td>
<td>0.9973</td>
<td></td>
<td></td>
<td></td>
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</table>
```
### Capability Indices

**Z Table:**

<table>
<thead>
<tr>
<th>Process Sigma Level</th>
<th>Process PPM</th>
<th>Process Sigma Level</th>
<th>Process PPM</th>
</tr>
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<tr>
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<td>6.00</td>
<td>3.4</td>
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<tr>
<td>6.12</td>
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<td>3.4</td>
<td>5.97</td>
<td>4</td>
</tr>
<tr>
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<td>5</td>
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<td>5</td>
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<td>40</td>
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<td>50</td>
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<td>60</td>
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<td>70</td>
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<td>90</td>
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<td>90</td>
<td>5.22</td>
<td>100</td>
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<tr>
<td>5.22</td>
<td>100</td>
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<td>400</td>
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</tr>
<tr>
<td>4.69</td>
<td>700</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Assumes that in the long term the process could drift by ± 1.5σ*
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Sigma Level (approach):
- Determine the proportions related to the upper & lower specifications.
- Calculate the proportion (defect rate).
- Calculate the DPMO.
- Use the Z Table to determine the equivalent Z Value to the DPMO.
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Example:

- 400 units were shipped, and 10 were returned as defective. Find out process Sigma Level:
  - Defect % = \( \frac{10}{400} \times 100\% = 2.5\% \)
  - Defect rate = \( \frac{10}{400} = 0.025 \)
  - DPMO = \( 1,000,000 \times 0.025 = 25,000 \)
  - Sigma Level = \( 3.46\sigma \) (from the Z Table)

Determining the Sigma Level allows process performance to be compared
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Normal distribution shifted 1.5σ

<table>
<thead>
<tr>
<th>SPEC LIMIT</th>
<th>PERCENT</th>
<th>DEFECTIVE PPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>±1 sigma</td>
<td>30.23</td>
<td>697,700</td>
</tr>
<tr>
<td>±2 sigma</td>
<td>69.13</td>
<td>308,700</td>
</tr>
<tr>
<td>±3 sigma</td>
<td>93.32</td>
<td>66,810</td>
</tr>
<tr>
<td>±4 sigma</td>
<td>99.379</td>
<td>6,210</td>
</tr>
<tr>
<td>±5 sigma</td>
<td>99.9767</td>
<td>233</td>
</tr>
<tr>
<td>±6 sigma</td>
<td>99.99966</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Normal distribution centred

<table>
<thead>
<tr>
<th>SPEC LIMIT</th>
<th>PERCENT</th>
<th>DEFECTIVE PPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>±1 sigma</td>
<td>68.27</td>
<td>317,300</td>
</tr>
<tr>
<td>±2 sigma</td>
<td>95.45</td>
<td>45,500</td>
</tr>
<tr>
<td>±3 sigma</td>
<td>99.73</td>
<td>2,700</td>
</tr>
<tr>
<td>±4 sigma</td>
<td>99.9937</td>
<td>63</td>
</tr>
<tr>
<td>±5 sigma</td>
<td>99.999943</td>
<td>0.57</td>
</tr>
<tr>
<td>±6 sigma</td>
<td>99.999998</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Actual / Long-term | Potential / Short-term
- Capability Indices

In control & capable of producing within control limits. A process with only natural causes of variation.

In control, but not capable of producing within control limits. A process with only natural causes of variation.

Out of control. A process out of control having assignable causes of variation.
- Capability Indices

Further Information:
- The 1.5 standards deviation value is a factor used to account for the shift and drift in the mean of a process output due to assignable causes over the long term.